Emission Control Policies and Endogenous Market Structure under Uncertainty: the Price vs. Quantity dilemma

Luca Di Corato
Università Ca' Foscari Venezia

Yishay D. Maoz
The Open University of Israel

July 2022

Abstract

In a competitive industry where production entails a negative externality, a welfare-maximizing regulator considers, as control instruments, setting a cap on the industry output or levying an output tax. We embed this scenario within a dynamic setup where market demand is stochastic and market entry is irreversible. We firstly determine the industry equilibrium under each policy and then determine the cap level and the tax rate which maximize welfare in each case. We show that a first-best outcome can be achieved through the tax policy while the cap policy may only qualify as a second-best alternative.

Keywords: Investment, Uncertainty, Caps, Taxes, Competition, Externalities, Welfare.

JEL codes: C61, D41, D62

Corresponding author: Yishay D. Maoz, Department of Management and Economics, The Open University of Israel, 1 University Road, Raanana, Israel. Email address: yishayma@openu.ac.il.
Luca Di Corato, Department of Economics, Ca' Foscari University of Venice, San Giobbe, Cannaregio 873, 30121 Venice, Italy. Phone: +390412347668. Email address: luca.dicorato@unive.it.

1. Introduction

This paper explores the regulation of market entries when external costs are associated with the production of commodities. This may be the case when, for instance, the entry of foreign firms negatively affects the domestic industry (Koenig, 1985), further market entries increase pollution (Spulber, 1985) or the development of new properties harm residents by reducing open space (Anderson, 1993).

Regulation may affect industry structure and equilibrium. In this respect, a relevant strand of literature[1] investigating the impact of environmental policy has shown, mostly using static models, that the internalization of the external cost generated by production depends on the degree of market competition. This is because the regulator must take into account the welfare losses that under imperfect competition may be due to distortions of the industry output and suboptimal market entries (see e.g. Spulber, 1985; Katsoulacos and Xepapadeas, 1995, 1996; Shaffer, 1995; Requate, 1997; Lee, 1999; Lahiri and Ono, 2007; Fujiwara, 2009). Another strand of literature has instead focused on the regulation of negative externalities in the framework of irreversible investment under uncertainty.[2] Baldursson and von der Fehr (1999) study the efficacy of price and quantity controls in a setup where the investment in abatement is irreversible for some firms in the industry and uncertainty is due to the entry and exit of polluting firms. Jou and Lee (2007) consider a real estate market assuming that newly developed properties, by reducing open space, have an external cost. They find that the internalization of such cost requires setting a positive tax on development or a negative tax on land value generate. In a similar framework, Lee and Jou (2007) show how the regulator can

[1]See Millimet et al. (2009) for a survey.
[2]Chao and Wilson (1993) show that the option value affects the investment in abatement under uncertain permit prices. Xepapadeas (1999) studies how a firm respond to environmental policy when deciding their investment in abatement and location under uncertainty about the output price, the policy context and the technology. Zhao (2003) shows that when considering uncertainty about the abatement costs the magnitude of the option value is larger when introducing taxes rather than permits.

correct the negative externality by imposing a density ceiling control. Di Corato and Maoz (2019) consider markets where production has adverse externalities and study the impact on welfare of a cap on the aggregate industry output. They include an external cost linearly increasing in the industry output in the welfare objective and set the cap endogenously. They show that a welfare-maximizing cap policy can only take one of the following two forms: (i) no cap at all; or (ii) immediately banning further market entries by setting the cap at the current industry output level. The choice depends on the level of uncertainty characterizing the firm's profits. In fact, when uncertainty is sufficiently high, no cap should be introduced since the uncertainty premium required for entering the market is such that in equilibrium the entry price threshold is above the social marginal cost of production and, therefore, any entry is always welfare increasing. In contrast, when the uncertainty premium is sufficiently small, banning further market entries is optimal since in equilibrium the entry price threshold is below the social marginal cost of production.

In this paper, we set up a model analyzing the industry equilibrium under perfect competition in a dynamic setup where market demand is stochastic and entry is irreversible. Production generates an external cost for Society which, differently than in Di Corato and Maoz (2019), is assumed increasing and convex in the industry output. We then consider the following two polar policy instruments for regulating the negative externality: (i) a quantity control exerted by introducing a cap on the industry output; (ii) a price control exerted by imposing an output tax.

Our main findings are as follows. As in Di Corato and Maoz (2019), we find that the presence of a cap does not affect the optimal entry strategy set at firm level, which remains identical to the one set in the absence of regulation.[3] Differently than in Di Corato and Maoz (2019), where the linearity of the external cost is crucial for their bang-bang result, we identify the circumstances under which an internal welfare-maximizing cap level exists. In this respect, we show that the key element is the gap between the marginal external cost and the marginal benefit, in terms of market surplus, associated with a new entry. If at the current industry output level, in response to further market entries, the external cost grows more than the market surplus, a ban should be imposed. Otherwise, an internal cap above the current industry output level should be set. Further, we find that the cap is increasing in the level of market uncertainty, a result that confirms the counterbalancing effect that, as shown by Di Corato and Maoz (2019), the uncertainty premium may have.

The analysis of the tax policy is straightforward. The emission tax can be viewed as an additional cost of production for the private firm and its impact can be studied using the model by Leahy (1993). In his model, the price threshold triggering market entries is increasing in the cost of production, therefore, the introduction of an emission tax, by raising the entry threshold, delays market entries with respect to the scenario where the industry is not regulated. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold. We then determine the tax rate maximizing welfare and find that it must be set equal to the marginal external cost associated with the industry output supplied at each time period. This implies that further market entries become less and less likely as the industry output increases since the higher the tax burden, the higher the entry threshold.

[3]The same result is obtained by Bartolini (1995) which considers the implications of a cap on the market output for firms' market entry and welfare However, the welfare analysis therein is not conclusive since the level of the cap is taken as exogenous and no external damages associated with firms' investment and production are explicitly considered.

Finally, when comparing the cap and the tax policies, an evident trade-off emerges. With the cap, the industry output is bounded but the cap does not affect its temporal evolution with respect to the scenario where the industry is not regulated. In contrast, with the emission tax, there is no limit to market entries but the tax affects the temporal evolution of the industry output by delaying market entries.

For a regulator maximizing welfare, a market entry is desirable as far as the associated gain in terms of market surplus covers its marginal social cost. In our set-up where firms may enter the market at any time point over an infinite time horizon, there is always a time point where this condition is met. Therefore, the ideal policy should be one able to delay market entries so that they occur at the "right" time from the regulator's perspective. Our analysis shows that this is feasible only via price control and, specifically, by equating the tax rate to the marginal external cost associated to the industry output supplied at each time point. This allows a complete internalization of the external cost by the firm when setting the entry strategy and, consequently, the industry equilibrium in our decentralized setting allows achieving a first-best outcome. In contrast, quantity control exerted through a cap policy may only qualify as a second-best alternative. This is because the resulting industry equilibrium is suboptimal for two reasons. First, the cap has no impact on the timing of market entries since firms keep setting their entry strategy without internalizing the associated external cost and, second, there is a loss of potential welfare gains associated with blocked market entries once the cap has been reached.

The paper remainder is as follows. In Section 2, we present our model set-up. In Section 3, we determine the industry equilibrium under no policy intervention. In Section 4, we introduce the two instruments for emission control and determine the optimal entry strategy under each policy. We determine the optimal cap and the optimal tax rate, compare the two polices and discuss our findings. Section 5 provides some remarks on our results, and Section 6 concludes.

2. The basic model

Within a continuous time setting, we consider a competitive industry comprised of a large number of identical firms that producing a certain good. Their individual size, *dn*, is infinitesimally small with respect to the market and they are all price takers.[4]

At each time point $t \geq 0$, the demand for this good is given by:

(1) $$P_t = X_t \cdot \phi(Q_t),$$

where P_t and Q_t are the market price and quantity of the good, respectively, $\phi(Q_t)$ is a deterministic component of the market demand with $\phi(Q_t) > 0$ and $\phi'(Q_t) < 0$ for any $Q_t > 0$, and $\lim_{Q_t \to \infty} \phi(Q_t) = 0$. The term X_t, is a demand shift factor that evolves stochastically over time according to the following Geometric Brownian Motion:

(2) $$dX_t = \mu \cdot X_t \cdot dt + \sigma \cdot X_t \cdot dZ_t,$$

where μ is the drift parameter, σ is the instantaneous volatility, and dZ_t is the increment of a standard Wiener process satisfying $E(dZ_t) = 0, \quad E(dZ_t)^2 = dt$ at each *t*.

[4] Assuming that firms are of infinitesimally small size is standard in models investigating the competitive equilibrium in a dynamic setting. See for instance Jovanovic (1982), Hopenhayn (1992), Lambson (1992), Leahy (1993), Dixit and Pindyck (1994, Ch. 8), Bartolini (1993, 1995) and Moretto (2008).

Each firm rationally forecasts the future evolution of the whole market. Market entry is free and an idle firm can enter the market at any time. By entering the market, the firm commits to permanently offer one unit of the good at each t. This implies that the industry output, Q_t, equals the number of active firms in the industry. Producing one unit of the good has a cost equal to $M > 0$.

Production entails a negative externality that firms do not incur. Its cost for Society, $D(Q_t)$, is a function of the industry output Q_t. We take the standard assumptions that $D'(Q_t) > 0$ and $D''(Q_t) > 0$ for any $Q_t > 0$ and $D(0) = 0$, implying that the external cost is positive, increasing and convex in the industry output.

Last, firms are risk-neutral profit maximizers and discount future payoffs using the interest rate r.[5] As standard in the literature, we assume that $r > \mu$ to secure that the firm's value is finite.

3. Industry equilibrium under no policy intervention

Let start by considering a scenario where no control policies are present. Under our model setup, a firm contemplating market entry is facing the same situation as the investors in Leahy (1993). Therefore, in the following, we use Leahy's analysis in order to determine the optimal entry strategy.[6]

At each time t, an idle firm has to decide whether to enter the market or not. By assumption, a firm entering the market commits to permanently produce one unit of the good at a cost equal to M. The present value of the associated flow of production costs, i.e. M/r, can be viewed as the irreversible investment that a firm must undertake in

[5] Note that introducing risk aversion would not affect our results, but merely require the development of the analysis under a risk-neutral probability measure. See Cox and Ross (1976) for further details.
[6] In the following, we will drop the time subscript for notational convenience.

order to enter the market. As future revenues are uncertain, market entry will occur when the expected profitability of such investment is sufficiently high.

Let $V(X,Q)$ be the value of an active firm given the current levels of *X* and *Q*. The standard no-arbitrage analysis in Appendix A shows that

$$V(X,Q) = Y(Q) \cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r}, \tag{3}$$

where $\beta > 1$ is the positive root of the quadratic equation

$$\tfrac{1}{2} \cdot \sigma^2 \cdot x^2 + \left(\mu - \tfrac{1}{2} \cdot \sigma^2\right) \cdot x - r = 0. \tag{3.1}$$

In (3), the term $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$ represents the expected present value of the flow of the firm's future profits conditional on *Q* remaining forever at its current level. Therefore, the first term, $Y(Q) \cdot X^{\beta}$, accounts for how future market entries reduce the value of the firm. This occurs since the firm's profit lowers as the industry output *Q* increases.

Two boundary conditions are required for finding the threshold function $X^*(Q)$ triggering market entry. The first one is the *Value Matching Condition*:

$$V\left[X^*(Q), Q\right] = 0, \tag{4}$$

and the second one is the *Smooth Pasting Condition*:

(5) $$V_X\left[X^*(Q),Q\right]=0\ .$$

Condition (4) is a standard zero-profit condition at the entry requiring that the value of an idle firm, which is null under free entry[7], must equal the value of an active one. Condition (5), in contrast, is an optimality condition that concerns the evolution of the demand shift, X_t, over time. Each time the process $\{X_t,t\geq 0\}$ hits the threshold $X^*(Q)$ a new firm enters the market and the price of the good, $P(Q)$, lowers since the supplied market quantity output has increased (see Dixit and Pindyck, 1994, Ch. 8, pp. 252-260).

Solving the system [4-5] yields the following result:

Proposition 1*:* Entry in a perfectly competitive market occurs every time the process $\{X_t,t\geq 0\}$ hits the threshold:

(6) $$X^*(Q)=\frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}}{\phi(Q)},$$

where $\hat{\beta}\equiv 1+\frac{1}{\beta-1}>1$.

Proof: Follows from applying (3) in (4) and (5). □

[7] The option to wait is valueless under free-entry since, as entry is attractive also for other firms, the firm, by postponing its entry, may lose out an investment opportunity (see Dixit and Pindyck, 1994, Ch. 8, pp. 256-258).

From $\phi'(Q_t)<0$ it follows that the threshold $X^*(Q)$ is an increasing function of Q, implying that the larger the market quantity supplied, the stronger the competition and then, ceteris paribus, the higher the profitability required for entering the market.

In time intervals where Q is not changed, the changes in X are translated, via (1), to changes in P. Based on standard properties of Brownian Motions, in such time intervals the proportional connection between X and P, as captured by (1), implies that P is also a Geometric Brownian Motion, and with the same parameters as X. On the other hand, at time instants when X hits the threshold function $X^*(Q)$, then a rise in X is not translated into a rise in P but leads to an increase in Q which keeps P unchanged. This occurs at the following level of P:

(7) $$P^* = X^*(Q)\cdot\phi(Q) = \hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r},$$

Which makes P^* an upper reflecting barrier regulating the process $\{P_t, t\geq 0\}$ and preventing the price from going above the level P^*. Figure 1 provides an illustration of these dynamics.

Note that, by the Marshallian rule, a firm should enter the market as long as $P = X\cdot\phi(Q)\geq(r-\mu)\cdot\frac{M}{r}$. Hence, by (6), the term $\frac{1}{\beta-1}>0$ (in $\hat{\beta}$) is the wedge by which the entry threshold should be adjusted in order to take the uncertainty and irreversibility into account (see Dixit and Pindyck, 1994, Ch. 5, Section 2). Last, note that $\frac{dP^*}{d\sigma^2}>0$ which follows from (7) taken together with the definition of $\hat{\beta}$ and also with $d\beta/d\sigma<0$ which is established in appendix A. This means that the higher the demand volatility, the higher the price threshold triggering firm's entry, which implies that market entry is delayed. This is because the output price, in its random evolution, needs more time (in expected terms) before hitting eventually a higher threshold.

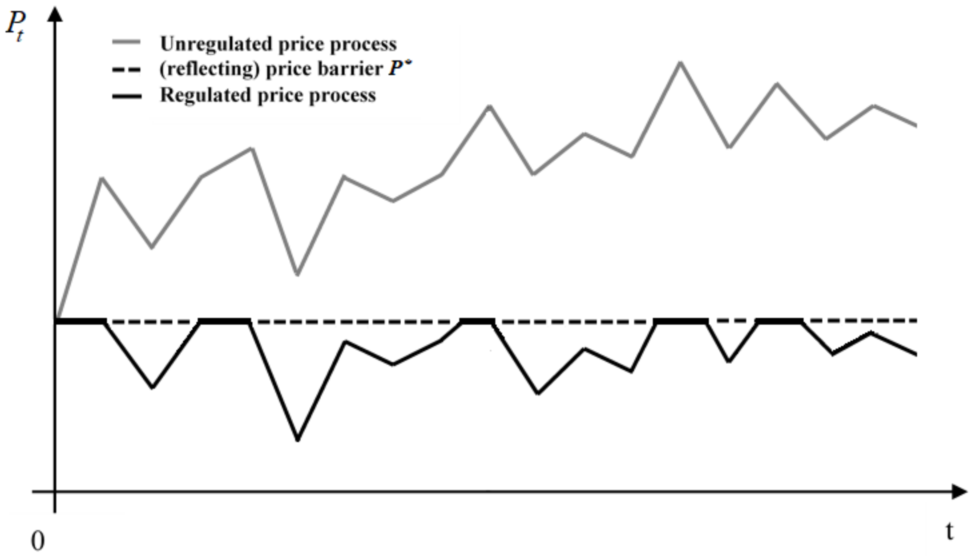

Figure 1: Price dynamics in a competitive industry

4. Industry equilibrium under policy intervention

The optimal entry strategy based on Eq. (6) does not account for the external cost associated with the negative externality that production entails once the firm has entered the market. In this section, we consider two policies for the reduction of the external cost: i) a cap on the industry output and ii) a tax on each unit of output. We first determine the industry equilibrium under each policy and then the level of the cap and the tax rate, respectively, maximizing welfare.

4.1 Industry equilibrium and welfare under a cap on the industry output

Assume that the government sets a cap on the aggregate industry output. Further, assume that entry licenses are distributed when the cap is announced. Each license allows producing one unit of output and their number is equal to difference between the cap, $\overline{Q}$, and the current level of the aggregate industry output, Q. We abstract from how the licenses are distributed since for our purposes their distribution has no other implications than providing to each firm owning a license the right to enter the market.[8]

4.1.1 The optimal entry strategy

The analysis of the firm's optimal entry under rationing is technically similar to the analysis in Section 3. The relevant difference between the two cases is that in this case the option to enter is an asset having a positive value that the firm gives up by entering the market. Thus, alongside the function $V(X,Q)$ which represents the value of an active firm, we define the function $F(X,Q)$ which stands for the value of the option to enter the market. A standard no-arbitrage analysis, similar to the one conducted in Appendix A for determining the value of an active firm, yields:

$$F(X,Q) = H(Q) \cdot X^{\beta}, \tag{8}$$

$$V(X,Q) = Y(Q) \cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r}, \tag{9}$$

where $H(Q)$ is to be found alongside the threshold $X^*(Q)$ by imposing the following *Value Matching Condition*:

[8] Note that, as shown by Bartolini (1995), the government may fully extract producer's surplus by allocating licenses through a competitive auction.

(10) $$V\left[X^*(Q),Q\right]=F\left[X^*(Q),Q\right],$$

and *Smooth Pasting condition*:

(11) $$V_X\left[X^*(Q),Q\right]=F_X\left[X^*(Q),Q\right].$$

Condition (10) asserts that the value of the option to enter, that is, the implicit cost of market entry, equals the value of an active firm, that is, the implicit return associated with market entry. Condition (11) secures optimality by imposing that the marginal cost of market entry equals its marginal return. As shown by Dixit (1993), Condition (10) holds for any entry threshold and merely reflects a no arbitrage assumption, while Condition (11) is an optimality condition which holds only at the optimal threshold.

Proposition 2*:* In a perfectly competitive market with cap on aggregate emissions, as long as the quantity in the market, *Q*, is below the cap, new entries to the market occur every time the process $\{X_t, t \geq 0\}$ hits the threshold:

(12) $$X^*(Q)=\frac{\hat{\beta}\cdot(r-\mu)\cdot\frac{M}{r}}{\phi(Q)},$$

or, equivalently, when the process $\{P_t, t \geq 0\}$ hits the barrier P^*, as captured by (7).

Proof: Follows from applying (8) and (9) in (10) and (11). □

Notably, the threshold function (12) does not depend on $\overline{Q}$ and is equal to the threshold function (6) determined under no policy intervention. The relevant difference here is that $X^*(Q)$ applies only until the cap $\overline{Q}$ is reached.

By Proposition (2) and (7), a new firm enters the market every time the process $\{P_t, t \geq 0\}$ hits the upper reflecting barrier P^*. As explained above, this prevents the price from going above the level P^*. However, under a cap policy, the regulation of the price through the barrier control applies only until the cap $\overline{Q}$ is reached and, once there, the output price starts moving freely over time following only the evolution dictated by (2). Figure 2 provides an illustration of these dynamics.

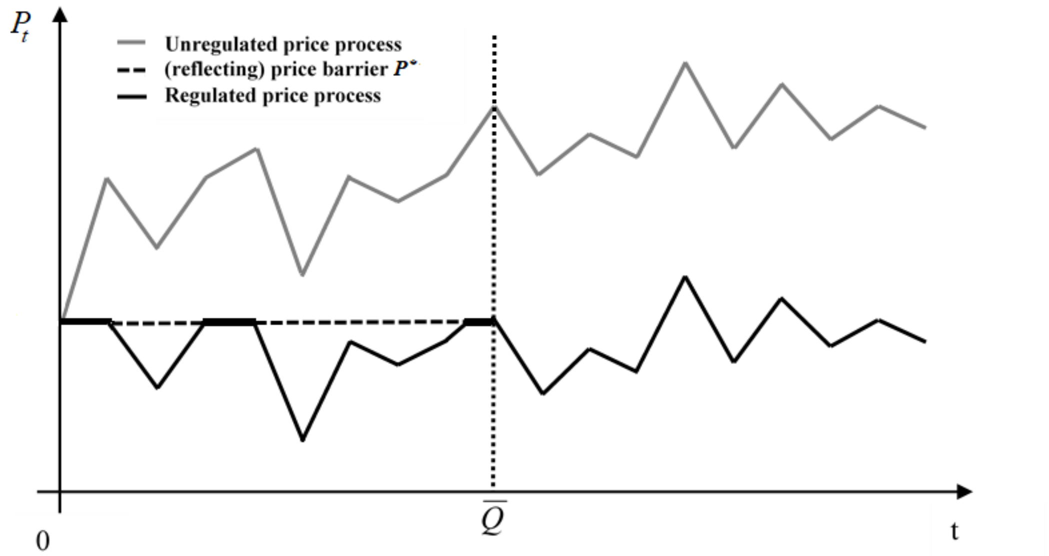

Figure 2: Price dynamics under a cap on the industry output

4.1.2 Welfare and the optimal cap

Once determined the industry equilibrium, in this section we determine the cap level maximizing welfare. This optimal level will trade off the welfare gains associated with lower negative externalities and the losses, in terms of market surplus, due to a lower quantity of the good available on the market once the cap has been reached.

Following a procedure similar to the one conducted in Appendix A for determining the value of an active firm, the expected discounted social welfare, given the current levels of *X*, and *Q* and the cap set at $\overline{Q}$, is:

(13) $$W\left(X, Q, \bar{Q}\right) = C\left(Q, \bar{Q}\right) \cdot X^{\beta} + \int_{0}^{Q} \left[\frac{P\left(X, q\right)}{r-\mu} - \frac{M + D'\left(q\right)}{r} \right] \cdot dq,$$

The integral in (13) represents the expected present value of welfare if the current industry output level, *Q*, will never change. For each unit supplied, the term $\frac{P(X,q)}{r-\mu}$ is the expected present value of the flow of market surplus associated with the supply of each unit of the good whereas the term $\frac{M+D'(q)}{r}$ is the present value of the flow of social costs associated to its production, i.e. private production costs plus external costs. The first term, $C\left(Q, \bar{Q}\right) \cdot X^{\beta}$, captures instead the contribution of future market entries to welfare.

At $X^*(Q)$ the following *Value Matching Condition* must hold:

(14) $$W_Q\left[X^*(Q), Q, \overline{Q}\right] = 0.$$

Condition (14) is a standard boundary condition stating that at each market entry the marginal welfare gain must equal the marginal welfare loss.

Further, at $Q = \bar{Q}$ we must impose that:

(15) $$C\left(Q, \bar{Q}\right) = 0.$$

The intuition behind Condition (15) is that the term $C\left(Q, \bar{Q}\right) \cdot X^{\beta}$ in (12) captures the welfare associated with future entries to the market. No such changes are possible once Q has reached the cap $\bar{Q}$ and thus $C\left(Q, \bar{Q}\right)$ must be null at $Q = \bar{Q}$.

Based on (13), (14) and (15) we show in Appendix B that:

(16) $$C\left(Q, \bar{Q}\right) = \int_{Q}^{\bar{Q}} \left[\frac{P^*}{r-\mu} - \frac{M + D'(q)}{r} \right] \cdot \frac{1}{X^*(q)^{\beta}} \cdot dq.$$

Differentiating $C\left(Q, \bar{Q}\right)$ with respect to $\bar{Q}$ yields:

(17) $$C_{\bar{Q}}\left(Q, \bar{Q}\right) = \left[\frac{P^*}{r-\mu} - \frac{M + D'\left(\bar{Q}\right)}{r} \right] \cdot \frac{1}{X^*\left(\bar{Q}\right)^{\beta}}.$$

(17) leads to the following proposition:

Proposition 3:

(a) If the current industry output level, Q, is sufficiently large so that $\frac{P^*}{r-\mu} \le \frac{M + D'(Q)}{r}$ then it is optimal to set the cap at the current Q, i.e., to immediately ban any further market entry.

(b) otherwise, if the current industry output level, Q, is sufficiently small so that $\frac{P^*}{r-\mu} > \frac{M+D'(Q)}{r}$ then the optimal level of the cap, denoted by $\overline{Q}^*$, is the root of the following equation:

$$\text{(18)} \qquad \frac{P^*}{r-\mu} = \frac{M + D'\left(\overline{Q}^*\right)}{r},$$

Proof: Follows from (17) and the convexity of $D(Q)$. □

By Proposition (3), if $\frac{P^*}{r-\mu} \le \frac{M+D'(Q)}{r}$, a ban deterring any further market entry is optimal. This is because the expected present value of the flow of market surplus added by the firm entering the market, i.e. $\frac{P^*}{r-\mu}$, does not cover the present value of the flow of social costs, i.e. $\frac{M+D'(Q)}{r}$, associated with the production of one more unit of the good. Otherwise, if $\frac{P^*}{r-\mu} > \frac{M+D'(Q)}{r}$, it is optimal setting a cap at a level higher than the current industry output level Q. Firms will then be allowed to enter the market until the industry output level $\overline{Q}^*$ is reached and where $\frac{P^*}{r-\mu} = \frac{M+D'\left(\overline{Q}^*\right)}{r}$.

Implicit differentiation of (18) yields that:

$$\text{(19)} \qquad \frac{d\overline{Q}^*}{d\sigma^2} = -\frac{1}{D''\left(\overline{Q}^*\right)} \cdot \frac{M}{(\beta-1)^2} \cdot \frac{d\beta}{d\sigma^2} > 0,$$

where the inequality follows from $D''(Q) > 0$, $\beta > 1$ and $\frac{d\beta}{d\sigma^2} < 0$. Thus, the higher the demand uncertainty the larger the optimal cap and the larger the industry output that the regulator is going to allow for. The reason for that is that a higher σ^2 leads, via its effect on the option wedge $\hat{\beta}$, to a higher P^* and, consequently, to a slower entry process in expected terms. This implies that while, on the one hand, the external cost increases at a slower speed, on the other hand, we incur into losses of market surplus since, having a higher entry barrier, P^*, market prices may reach relatively higher levels before a new firm enters the market. Further, one must account for the fact that, even tough, once reached the cap, the external cost stops increasing, there is a loss of market surplus due to the fact that, as no firms may enter the market, the output price evolves freely being absent the barrier control preventing it from going above the level P^*. The loss of market surplus may be relevant and consistently,

(20) $$\lim_{\sigma^2 \to \infty} \overline{Q}^* = \infty,$$

which means that setting an internal cap, $\overline{Q}^*$, is not optimal since restricting firms' entry is too costly in the presence of high levels of market uncertainty.

Implicit differentiation of (18), also yields:

(21) $$\frac{d\overline{Q}^*}{dM} = \frac{1}{D''\left(\overline{Q}^*\right)} \cdot \frac{1}{\beta - 1} > 0,$$

where the inequality follows from $D''(Q) > 0$ and $\beta > 1$. Thus, the higher the production cost the larger the optimal cap and therefore the larger the market size that the regulator is going to allow for. The reason for that is that the larger *M*, the higher the price that triggers entry, i.e. P^*, and, consequently, the slowest the entry process in expected terms. This has, as above, implications for the speed at which the external cost increases and the magnitude of the flow of market surplus.

Last, based on *Proposition 3* and (13), in the case where the optimal cap is at the current *Q*, the expected discounted social welfare is equal to:

$$W^{cap}(X,Q)=\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq\,, \tag{22}$$

otherwise, when the optimal cap is $\overline{Q}^*$, the expected discounted social welfare is:

$$W^{cap}(X,Q)=\int_Q^{\overline{Q}^*}\left[\frac{P^*}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot\left[\frac{X}{X^*(q)}\right]^\beta\cdot dq +\int_0^Q\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(q)}{r}\right]\cdot dq \tag{23}$$

As Dixit and Pindyck (1994, page 315) show, the term $\left[\frac{X}{X^*(q)}\right]^\beta$ is equal to the discount factor $E\left[e^{-r\cdot T(q)}\right]$, where $T(q)$ is the time when process $\{X_t,\, t > 0\}$, starting from its current level (X), hits the threshold level $X^*(q)$ for the first time. This insight enables the following rather intuitive view of the resulting formula for the welfare function, as captured by (23):

- The last term, $\int_0^Q \left[\frac{P(X,q)}{r-\mu} - \frac{M+D'(q)}{r} \right] \cdot dq$, is an integral over the already existing units of the expected present value of the flow of social welfare (price less costs) expected from each such unit.
- The first integral, therefore is the welfare value of future entries, those that will add units from the current quantity, Q, to the maximum allowed by the cap $\bar{Q}^*$. The welfare value of each such future units comprises two parts:

    - $\frac{P^*}{r-\mu} - \frac{M+D'(q)}{r}$, which is the expected value of the flow of welfare that this unit would yield, from the moment in which it is installed and given that it is installed when the market price is P^*.
    - $\left[\frac{X}{X^*(q)} \right]^{\beta}$, is the expected value of the discount factor of the welfare flow that would spring from this unit.

4.2 Industry equilibrium and welfare under an output tax

Assume that the government levies a tax $\tau > 0$ per unit of output. The analysis of the industry equilibrium under an output tax is technically identical to the one conducted in Section 3. The only difference is that here the cost for producing one unit of output is equal to $M + \tau$. Hence:

Proposition 4*:* Entry in a perfectly competitive market under an output tax occurs every time the process $\{X_t, t \geq 0\}$ hits the threshold:

$$(24) \qquad X^{**}(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \frac{M+\tau}{r}}{\varphi(Q)} > X^*(Q),$$

or, equivalently, the process the $\{P_t, t \geq 0\}$ hits the barrier:

(25) $$P^{**} = \hat{\beta} \cdot (r - \mu) \cdot \frac{M + \tau}{r} > P^{*}$$

Proof: Follows from repeating the proof of *Proposition 1*, this time with a private production cost equal to $M + \tau$. □

By (24), the entry threshold is increasing in the tax rate τ, which implies, in expected terms, that market entries are slower than in the case of a cap. This implies that, on the one hand, the external cost increases at a slower speed, while, on the other hand, each market entry, even though delayed, pays a higher market surplus since, by (25), $\frac{P^{**}}{r-\mu} > \frac{P^{*}}{r-\mu}$ for any Q. Figure 3 provides an illustration of these dynamics.

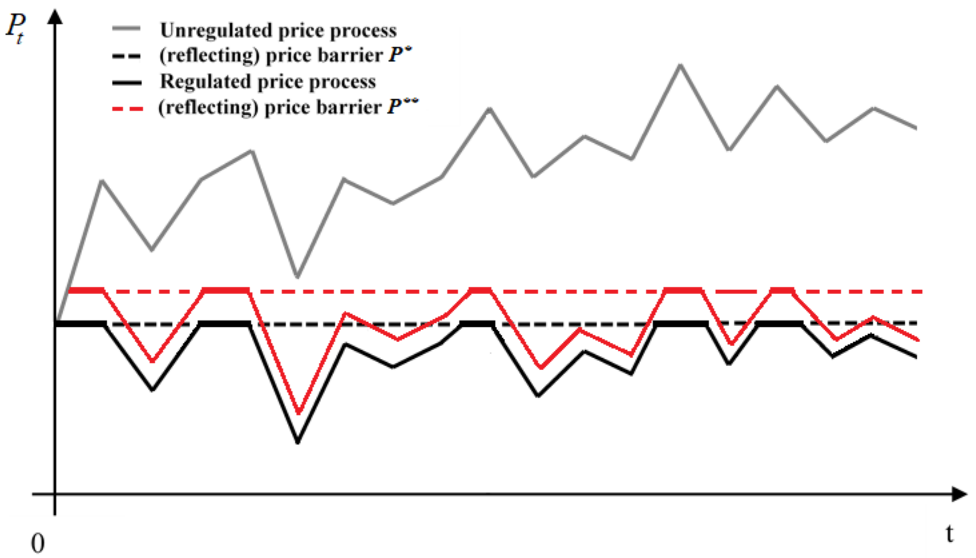

Figure 3: Price dynamics under an output tax

4.2.1 Welfare and the optimal tax rate

The expected discounted social welfare, given the current levels of X and Q, is:

(26) $$W(X,Q,\tau)=C(Q,\tau)\cdot X^{\beta}+\int_{0}^{Q}\left[\frac{P(X,q)}{r-\mu}-\frac{M+D'(Q)}{r}\right]\cdot dq.$$

The transfers of tax payments from producers to the government lower the producers' profits and raise the government revenues by the same amount and therefore cancel out of the social welfare. Thus, the only remaining channel by which taxes affect social welfare is via their effect on the firms' entry thresholds and therefore on entry times. Thus, in setting an optimal tax policy, the government is in fact setting an optimal threshold policy. We find the optimal entry threshold policy via the methodology presented by Dixit and Pindyck (1994, page 286) for that end. Specifically, the analysis is based on the following two boundary conditions: The *Value Matching Condition*:

(27) $$W_{Q}\left[X^{**}(Q),Q\right] =0,$$

and *Smooth Pasting Condition*:

(28) $$W_{QX}\left[X^{**}(Q),Q\right] =0.$$

Similar to the welfare analysis in Section 3, condition (27) is not an optimality condition but merely a no-arbitrage condition that holds for any entry threshold, not necessarily the optimal one. In contrast, Condition (28) is an optimality condition that leads to the entry threshold which is optimal from the regulator's perspective and to the tax rate that leads to this optimal threshold.

Applying (26) in (27) and (28) yields that the optimal threshold is:

(29) $$X^{**}(Q) = \frac{\hat{\beta} \cdot (r-\mu) \cdot \frac{M + D'(Q)}{r}}{\varphi(Q)},$$

which leads to the following proposition regarding the optimal tax:

Proposition 5*:* The welfare maximizing tax rate is:

(30) $$\tau^* = D'(Q),$$

Proof: Follows directly from comparing (24) and (29). □

Notably, by substituting (30) into (24) and rearranging, one may easily show that

(31) $$\frac{P^{**}}{r-\mu} = \hat{\beta} \cdot \frac{M + D'(Q)}{r} > \frac{M + D'(Q)}{r}, \text{ for any given } Q.$$

By (31), market entries are always beneficial since the expected present value of the flow of market surplus added by a new firm entering the market, i.e. $\frac{P^{**}}{r-\mu}$, covers always the present value of the flow of social costs, i.e. $\frac{M + D'(Q)}{r}$, associated with the production of one more unit of the good. This is because at each entry the barrier level P^{**} is adjusted upward by taxing at a tax rate τ^* which is increasing in *Q*. Therefore, market entries occur always at a price which is sufficiently high to secure, once accounted for the added external cost, a positive contribution to welfare. Figure 4 provides an illustration of these dynamics.

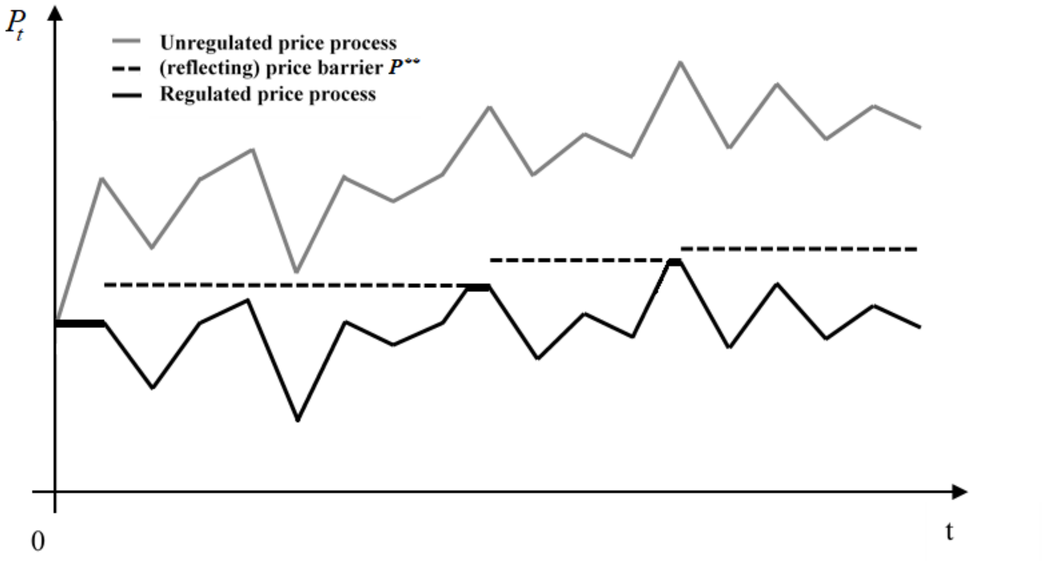

Figure 4: Price dynamics under optimal output taxation

Conditions (27) and (28) also yield:

(32) $$C_Q\left(Q,\tau^*\right) = -\left[\frac{P^{**}}{r-\mu} - \frac{M + D'(Q)}{r}\right] \cdot \frac{1}{X^{**}\left(Q,\tau^*\right)^{\beta}}$$

To integrate (32) we use the following boundary condition:

(33) $$\lim_{Q\to\infty} C(Q,\tau) = 0.$$

The intuition behind Condition (33) is immediate. In (26), the term $C(Q,\tau)\cdot X^{\beta}$ captures the welfare associated with future increases of the industry output. No such changes are expected when $Q \to \infty$ because in that case the entry threshold (24) goes to infinity by the assumption $\lim_{Q\to\infty} \phi(Q) = 0$.

Integrating (32) and applying (33) yields:

$$C\left(Q, \tau^*\right) = \int_Q^\infty \left[\frac{P^{**}}{r-\mu} - \frac{M + D'(q)}{r}\right] \cdot \frac{1}{X^{**}\left(q, \tau^*\right)^\beta} \cdot dq \,. \tag{34}$$

Applying (34) and (1) in (26) yields that the expected discounted social welfare when the tax rate is optimally set is equal to:

$$\begin{aligned} W^{tax}(X, Q) = {} & \int_Q^\infty \left[\frac{P^{**}}{r-\mu} - \frac{M + D'(q)}{r}\right] \cdot \left[\frac{X}{X^{**}\left(Q, \tau^*\right)}\right]^\beta \cdot dq + \\ & + \int_0^Q \left[\frac{P(X, q)}{r-\mu} - \frac{M + D'(q)}{r}\right] \cdot dq \end{aligned} \tag{34}$$

4.3 Comparing industry equilibrium and welfare under the two policies

In this Section, we compare the two policies from a welfare-maximizing perspective and find that the tax policy is better than the cap policy. In fact, the optimal tax policy we have found in the previous sub-section for a decentralized case where the government chooses a tax policy is also the first-best public policy that a central planner would choose. More specifically, consider a central planner that can choose all allocations, and maximizes welfare given preferences and technology, including the irreversibility limitation. For this social planner, the relevant allocations to set is the incremental additions to quantity, and specifically what the central planner chooses is when to add each increment. In that sense this case completely coincides with the decentralized case of a government choosing an optimal tax policy studied in the previous sub-section, as the only effect taxes have on welfare is via the entry times of the competitive firms.

This intuition suggests that the tax policy, being a first-best, should yield greater welfare then the cap policy. The following *Proposition 6* asserts it.

Proposition 6*:* The welfare achieved by the optimized tax policy, as captured by (34) exceeds the welfare achieved by the optimized cap policy, as captured by (23).

Proof: See appendix C. □

5. Final remarks

Some final remarks are in order:

1) assume that the regulator announces a cap $\overline{Q}$ on aggregate emissions and impose that each firm may produce not more than $0 < \lambda < 1$ units. As one may immediately see, introducing this variation in our model set-up would have no impact on our results. The only thing that one should keep in mind is that in this case i) the number of active firms in the industry is equal to Q/λ and ii) the maximum number of firms entering the market is equal to $\overline{Q}/\lambda$;

2) in Section 4.1 we have assumed that entry licenses are distributed when the cap is announced. In the presence of entry licenses, firms holding a license may optimally exercise their option to invest since the threat of being preempted by others is absent. Otherwise, if firms are not licensed, they will gradually enter the market by (12) only

up to a certain time point, then a competitive run will start and the cap will be instantly reached (see Bartolini, 1993 and 1995). In this case, as shown by Di Corato and Maoz (2019) in a similar frame, welfare is always lower than under rationing. This implies that our main results would be even stronger.

3) in a centrally planned framework, Weitzman (1974) concludes that with uncertain benefit and/or cost function, quantity control (price control) performs better if and only if the marginal benefit (cost) curve is steeper than the marginal cost (benefit) curve. In contrast, we show that, irrespective of the shape of benefit and cost curves, i.e. reduced external cost and lower market surplus, respectively, price control (through the emission tax) performs always better than quantity control (through the cap).

6. Conclusions

In this paper, we have presented a model of endogenous market structure under uncertainty, with production externalities regulated by a cap on the industry output or via an output tax. The main result is that the tax policy dominates the cap policy when aiming at the maximization of the welfare. In particular, we show that the tax policy allows achieving a first-best outcome since the external cost associated with production is fully internalized. We are aware that, concerning the complete internalization of the external cost, the assumption of perfectly competitive firms is crucial. It becomes then of interest, as potential lead for future research, extending the analysis in order to consider the impact that market power, by distorting the industry output, have on the degree of internalization and, potentially, on how the two policies should be ranked from a welfare-maximizing perspective.

APPENDIX

Appendix A – The value of an active firm

In this Appendix, we present the derivation of the value function in (3), i.e. $V(X,Q)$. By a standard no-arbitrage argument (see e.g. Dixit, 1989), $V(X,Q)$ is the solution of the following Bellman equation:

$$\text{(A.1)} \qquad r \cdot V(Q, X) \cdot dt = \left[P(X,Q) - M\right] \cdot dt + E\left[dV(X,Q)\right],$$

which states that the instantaneous profit, $\left[P(X,Q) - M\right] \cdot dt$, along with the expected instantaneous capital gain, $E\left[dV(X,Q)\right]$, from a change in *X*, must be equal to the instantaneous normal return, $r \cdot V(X,Q) \cdot dt$.

Itô's lemma states that since *X* is a geometric Brownian motion with parameter μ and σ then $V(X,Q)$, being a twice differentiable function of *X* satisfies:

$$\text{(A.2)} \qquad dV(X,Q) = \left[\tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X,Q) + \mu \cdot X \cdot V_X(X,Q)\right] \cdot dt + \sigma \cdot X \cdot dZ .$$

Applying (A.2) in (A.1), taking the expectancy recalling that E(*dZ*) = 0, and rearranging, yields:

$$\text{(A.3)} \qquad \tfrac{1}{2} \cdot \sigma^2 \cdot X^2 \cdot V_{XX}(X,Q) + \mu \cdot X \cdot V_X(X,Q) - r \cdot V(X,Q) + P(X,Q) - M = 0 .$$

Trying a solution of the type x^b for the homogenous part of (A.3) and a linear form as a particular solution to the entire equation yields:

(A.4) $$V(X,Q) = Z(Q)\cdot X^{\alpha} + Y(Q)\cdot X^{\beta} + \frac{P(X,Q)}{r-\mu} - \frac{M}{r},$$

where $\alpha < 0$ and $\beta > 1$ are the roots of the quadratic equation:

(A.5) $$\tfrac{1}{2}\cdot\sigma^2\cdot x\cdot(x-1) + \mu\cdot x - r = 0.$$

Applying $x = 0$ and then $x = 1$, and bearing in mind that $r > \mu$ asserts that (A.5) has two roots, one of them negative and the other exceeds 1.

The first term in (A.4), namely $\frac{P(X,Q)}{r-\mu} - \frac{M}{r}$, represents the expected present value of the flow of profits conditional on Q remaining forever at its current level. Therefore, the first and second term on the RHS of (A.3) should capture the impact that changes in Q over time have on the value of the firm in expected terms.

By the properties of the Geometric Brownian Motion, when X goes to 0 the probability of ever hitting the barrier triggering a new entry, i.e., $X^*(Q)$, and, consequently, an increase in Q, tends to 0. This leads to the following limit condition:

(A.6) $$\lim_{X\to 0}\left[Z(Q)\cdot X^{\alpha} + Y(Q)\cdot X^{\beta}\right] = 0.$$

Note that as α < 0, (A.6) holds only if $Z(Q) = 0$ for any $Q > 0$. Hence, substituting $Z(Q) = 0$ into (A.3) gives (3).

Finally, applying β for x in (A.5) leads to:

(A.6) $$\frac{d\beta}{d\sigma^2} = -\frac{\frac{1}{2}\cdot\beta\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot(2\cdot\beta-1)+\mu} = -\frac{\frac{1}{2}\cdot\beta^2\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot\beta^2+\frac{1}{2}\cdot\sigma^2\cdot\beta\cdot(\beta-1)+\mu\cdot\beta}$$

$$= -\frac{\frac{1}{2}\cdot\beta^2\cdot(\beta-1)}{\frac{1}{2}\cdot\sigma^2\cdot\beta^2+r} < 0\,,$$

where the third equality follows from (A.4), evaluated at β, and the inequality springs from $\beta > 1$.

Appendix B – Welfare maximization under a cap on aggregate emissions

Substituting the derivative of (13) with respect to Q in (14), applying (12), and rearranging terms, yields:

(B.1) $$C_Q\left(Q,\bar{Q}\right) = -\left[\frac{P^*}{r-\mu} - \frac{M+D'\left(\bar{Q}\right)}{r}\right]\cdot\frac{1}{X^*\left(Q\right)^{\beta}}\,,$$

Integrating (B.1) yields:

(B.2) $$C\left(\bar{Q},\bar{Q}\right) - C\left(Q,\bar{Q}\right) = -\int_{Q}^{\bar{Q}}\left[\frac{P^*}{r-\mu} - \frac{M+D'\left(\bar{Q}\right)}{r}\right]\cdot\frac{1}{X^*\left(Q\right)^{\beta}}\cdot dq\,.$$

The term $C\left(Q,\overline{Q}\right)\cdot X^{\beta}$ in (13) captures the welfare associated with future increases of the industry output. No such changes are possible if Q has reached the cap $\overline{Q}$. Therefore, the following boundary condition holds at $Q = \overline{Q}$:

(B.3) $$C\left(\bar{Q}, \bar{Q}\right) = 0,$$

Substituting (B.3) in (B.2) yields:

(B.4) $$C\left(Q, \bar{Q}\right) = \int_{Q}^{\bar{Q}} \left[\frac{P^*}{r-\mu} - \frac{M + D'\left(\bar{Q}\right)}{r} \right] \cdot \frac{1}{X^*\left(Q\right)^{\beta}} \cdot dq \, .$$

Appendix C – Proof of Proposition 6

In this appendix, we show that welfare under an output tax exceeds welfare under a cap on the industry output. The proof is as follows:

(C.1) $$W^{tax}\left(X, Q^*\right) - W^{cap}\left(X, Q\right)$$

$$= \int_{Q}^{\infty} \frac{\left[M + D'(q)\right] \cdot X^{\beta}}{(\beta-1) \cdot r \cdot \left[X^{**}\left(q, \tau^*\right)\right]^{\beta}} \cdot dq - \int_{Q}^{\bar{Q}^*} \frac{\left[M - (\beta-1) \cdot D'(q)\right] \cdot X^{\beta}}{(\beta-1) \cdot r \cdot \left[X^*(q)\right]^{\beta}} \cdot dq$$

$$> \int_{Q}^{\bar{Q}^*} \frac{M + D'(q)}{(\beta-1) \cdot r} \cdot \left[\frac{X^*(q)}{X^{**}\left(q, \tau^*\right)} \right]^{\beta} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq$$

$$- \int_{Q}^{\bar{Q}^*} \frac{M + D'(q) - \beta \cdot D'(q)}{(\beta-1) \cdot r} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq$$

$$= \int_{Q}^{\bar{Q}^*} \left[M + D'(q)\right] \cdot \frac{\left[h(q)\right]^{\beta} - 1 + \beta \cdot \left[1 - h(q)\right]}{(\beta-1) \cdot r} \cdot \left[\frac{X}{X^*(q)} \right]^{\beta} \cdot dq > 0$$

where the first equality follows from (22) and (39); the first inequality follows from narrowing the range over which the first integral goes (noticing from *Proposition 3* that the integrand is positive in that range); the second equality follows from defining:

(C.2) $$h(q) \equiv \frac{X^*(q)}{X^{**}\left(q, \tau^*\right)} = \frac{M}{M + D'(q)},$$

which follows from (12) and (33) and also leads to $1 - h(q) = \frac{D'(q)}{M + D'(q)}$. From $D'(q) > 0$ it follows that $0 < h(q) < 1$ for any $q > 0$.

The last inequality in (C.1) holds because any function of the form $g(x) \equiv x^\beta - 1 + \beta \cdot (1 - x)$ is positive within the range $0 < x < 1$, as:

- $g(0) = \beta - 1 > 0$
- $g(1) = 0$
- $g'(x) = \beta \cdot \left(x^{\beta - 1} - 1\right) < 0$ for all $0 < x < 1$.

References

Anderson, J.E., 1993. "Land development, externalities, and Pigouvian taxes." *Journal of Urban Economics*, 33, 1-9.

Baldursson, M.F., von der Fehr, N.-H.M., 2004. "Prices vs. quantities: the irrelevance of irreversibility". *Scand. J. of Economics*, 106, 805-821.

Bartolini, L., 1993. "Competitive runs: The case of a ceiling on aggregate investment." *European Economic Review*, 37(5), 921-948.

Bartolini, L., 1995. "Foreign investment quotas and rent extraction under uncertainty." *Journal of International Economics*, 38, 25-49.

Chao, H.P., Wilson, R., 1993. “Option value of emission allowances.” *Journal of Regulatory Economics*, 5, 233-249.

Cox, J.C., Ross, S.A., 1976. “The valuation of options for alternative stochastic processes.” *Journal of Financial Economics,* 3, 145-166.

Di Corato, L., Maoz, Y. D., 2019. “Production externalities and investment caps: A welfare analysis under uncertainty.” *Journal of Economic Dynamics & Control*, 106, September 2019, 103719.

Dixit, A.K., 1989. “Entry and exit decisions under uncertainty.” *Journal of Political Economy*, 97, 620-638.

Dixit, A.K., 1993. “The Art of Smooth Pasting.” Harwood Academic Publishers.

Dixit, A.K., Pindyck, R.S., 1994. “Investment and Uncertainty.” Princeton University Press, Princeton.

Fujiwara, K., 2009. “Environmental policies in a differentiated oligopoly revisited.” *Resource and Energy Economics*, 31, 239-247.

Hopenhayn, H., 1992. “Entry, exit and firm dynamics in long run equilibrium.” *Econometrica*, 60, 1127-1150.

Jou, J.-B., Lee, T., 2008. “Taxation on Land Value and Development When There Are Negative Externalities from Development.” *The Journal of Real Estate Finance and Economics*, 36, 103-120.

Jovanovic, B., 1982. “Selection and the evolution of industry.” *Econometrica*, 50, 649-670.

Katsoulacos, Y., Xepapadeas, A., 1995. “Environmental Policy under Oligopoly with Endogenous Market Structure.” *The Scandinavian Journal of Economics*, 97, 411-420.

Katsoulacos, Y., Xepapadeas, A., 1996. "Emission taxes and market structure." In Carraro, C., Katsoulacos, Y., Xepapadeas, A. (Eds.), Environmental Policy and Market Structure, Kluwer, Boston (1996), 3-22.

Koenig, E.F., 1985. "Indirect Methods for Regulating Externalities Under Uncertainty." *The Quarterly Journal of Economics*, 100, 479-493.

Lahiri, S., Ono, Y., 2007. "Relative emission standard versus tax under oligopoly: the role of free entry." *Journal of Economics*, 91, 107-28.

Lambson, V.E., 1992. "Competitive profits in the long run." *Review of Economic Studies*, 59, 125-142.

Leahy, J., 1993. "Investment in competitive equilibrium: the optimality of myopic behavior." *Quarterly Journal of Economics*, 108, 1105-1133.

Lee, S-H., 1999. "Optimal taxation for polluting oligopolists with endogenous market structure." *Journal of Regulatory Economics*, 15, 293-308.

Lee, T., Jou, J.-B., 2007. "The regulation of optimal development density." *Journal of Housing Economics*, 16, 21-36.

Millimet, D.L., Roy, S., Sengupta, A., 2009. "Environmental regulations and economic activity: influence on market structure." *Annual Review of Resource Economics*, 1, 99-117.

Moretto, M., 2008. "Competition and irreversible investment under uncertainty." *Information Economics and Policy*, 20, 75-88.

Requate, T., 1997. "Green taxes in oligopoly if the number of firms is endogenous." *Finanzarchiv*, 54, 261-280.

Shaffer, S., 1995. "Optimal Linear Taxation of Polluting Oligopolists." *Journal of Regulatory Economics*; 7, 85-100

Spulber, D., 1985. “Effluent regulation and long run optimality”. *Journal of Environmental Economics and Management*. 12:103-116.

Weitzman, M. L., 1974. “Prices vs. quantities.”. *The Review of Economic Studies*, 41, 477-491.

Xepapadeas, A., 1999. “Environmental policy and firm behavior: abatement investment and location decisions under uncertainty and irreversibility.” Working Paper T0243, NBER.

Zhao, J., 2003. “Irreversible abatement investment under cost uncertainties: tradable emission permits and emission charges.” *Journal of Public Economics*, 87, 2765-2789.